TIME SERIES 501

Lesson 4: AR-MA

Learning Objectives

You will be able to do the following:

- Describe the autocorrelation function (ACF).
- Describe the partial autocorrelation function (PACF).
- Explain how autoregressive and moving average models work.
- Use Python* to fit autocorrelation models.

ACF AND PACF

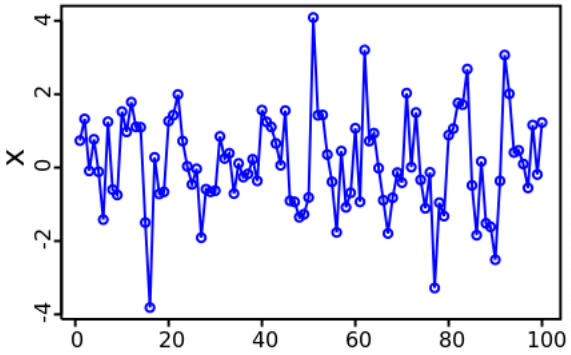
Autocorrelation Function (ACF)

- Measures the correlation of a signal with a delayed copy of itself.
- It is used to find repeating patterns in a signal, such as the presence of a periodic signal.

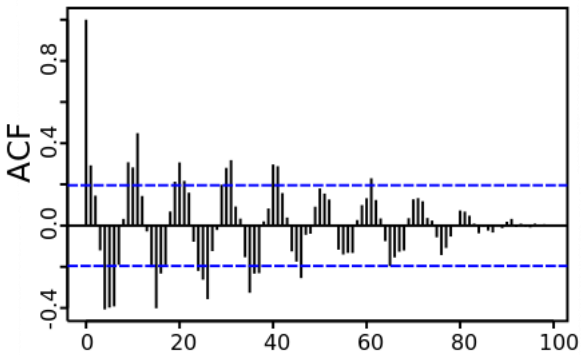
$$R(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2}$$

where μ is the mean and σ^2 is the variance

Autocorrelation Function (ACF)



Observed Data



ACF Plot

https://commons.wikimedia.org/wiki/File:Acf_new.svg

Partial Autocorrelation Function

The partial autocorrelation at lag k is the autocorrelation between X_t and X_{t-k} that is not accounted for by lags 1 through k-1

$$\alpha(1) = Cor(x_{t+1}, x)$$

$$\alpha(k) = Cor\left(x_{t+k} - P_{t,k}(x_{t+k}), x_t - P_{t,k}(x_t)\right)$$

where $P_{t,k}(x)$ denotes the projection of x onto the space spanning x_{t+1}, ... x_{t+k-1}

AUTOREGRESSIVE MODELS

Autoregressive Models (AR)

- A common approach to model univariate time series is to use autoregressive models (AR).
- An AR model is a linear regression of the current value of the series against one or more prior values of the series.
- Uses maximum likelihood estimators to determine coefficients instead of least squares.

Autoregressive Models (AR)

The notation AR(p) indicates an autoregressive model of order p.

$$X_t = \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t$$

where φ_i are the parameters of the model and ε_t is white noise.

Autoregressive Models (AR)

- Useful for short term forecasts
- This model can forecast up to p values

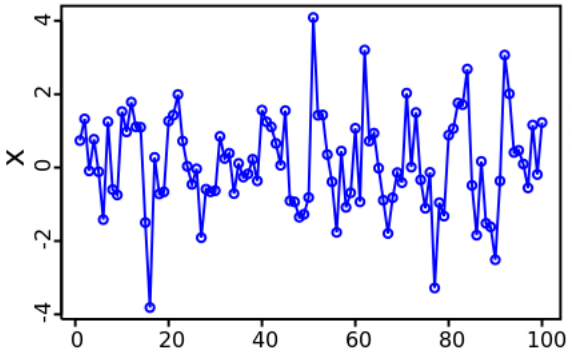
Steps for Forecasting with AR

Step 1: Model the trend and/or seasonality of a time series and subtract it from the data (X_t = Observed$_t$ – Trend$_t$ – Seasonality$_t$)

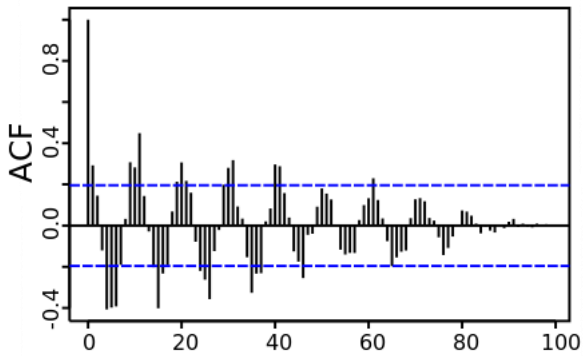
Step 2: Calculate AR on X_t to obtain X_{t+1}

Step 3: Forecast = X_{t+1} + Trend$_t$ + Seasonality$_t$

ACF Plot Details



Observed Data

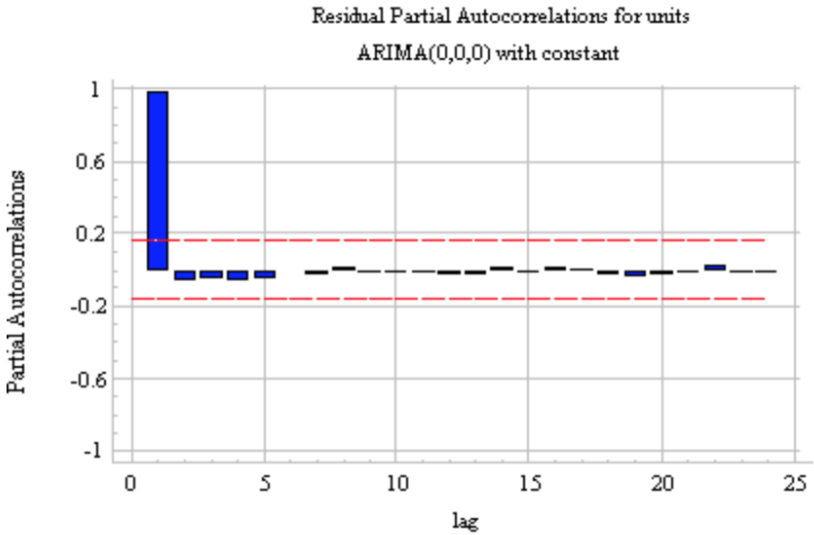


ACF Plot

https://commons.wikimedia.org/wiki/File:Acf_new.svg

PACF Plot

- Let's say we generated the PACF plot to the right.
- We see a single significant spike at lag 1.
- This means that all other significant spikes in the ACF plot are explained by the spike at lag 1.
- Therefore, we would choose the model AR(1).



PACF Plot

Source: https://people.duke.edu/~rnau/411arim3.htm

MOVING AVERAGE MODELS

Moving Average Models (MA)

- Another common approach to modeling univariate time series is the moving average (MA) model.
- MA models are conceptually a linear regression of the current value of the series against the white noise of one or more of the previous values of the series.
- The noise at each point is assumed to come from a normal distribution with mean 0 and constant variance.

Moving Average Models (MA)

- Fitting MA estimates is more complicated than AR models because the error terms are not observable.
- As a result, iterative nonlinear fitting procedures need to be used.
- MA models are less interpretable than AR models.

Moving Average Models (MA)

The notation MA(q) indicates a moving average model of order q.

$$X_t = \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

where θ_i are the parameters of the model and ε_t is white noise.

Moving Average Models (MA)

- We talked about smoothing with a moving average in Lesson 3.
- Moving average models are not the same as smoothing.
- Please don't confuse the two.

AR or MA Model?

- If the **PACF drops sharply** at a given lag or the first lag autocorrelation is **positive**, then use an **AR model** with order p equal to the lag just before the sharp decline.

- If the **ACF drops sharply** at a given lag or the first lag autocorrelation is **negative**, then use an **MA model** with order q equal to the lag just before the sharp decline.

APPLICATIONS IN PYTHON

Use Python to Fit Autocorrelation Models

Next up is a look at applying these concepts in Python.

- See notebook entitled *Autocorrelation_student.ipynb*

Learning Objectives Recap

In this session you learned how to do the following:

- Describe the autocorrelation function (ACF)
- Describe the partial autocorrelation function (PACF).
- Explain how autoregressive and moving average models work.
- Use Python to fit autocorrelation models.

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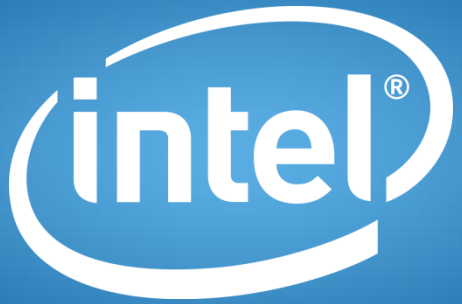
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